A Kohonen map for Chury

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Abstract

The highly irregular shape of Rosetta's target comet 67P/Churyumov-Gerasimenko (or "Chury" for short) poses some challenges for mapping, in particular for displaying the complete comet in one map. Global map projections for the Earth (and other — more or less — spherical solar system bodies) rely on the requirement that a surface point can uniquely be identified by longitude and latitude. However, because of the large overhanging areas, there are ranges of longitude and latitude for which there are three different surface points. Therefore, a significant area of Chury is not visible in a map resultant from the naive application of any standard global projection.

A so called Kohonen map is a self-adapting artificial neural network that allows it to fit a rectangular grid to any kind of data, including a closed three-dimensional surface. While this approach is straight forward, there is a shortcoming: the grid wraps itself around the shape, trying to cover it as evenly as possible with all its grid points, but there is always an uncovered gap where the edges of the grid approach each other. This gap gets narrower with increasing grid resolution, but it never closes completely.

Here, we create an inherently closed structure by taking two square grids and "stitching" them together at all four edges. This closed Kohonen map is then fitted to the comet shape in the same way as the simple open map, yielding a complete map of the whole surface. Because of its construction from two squares, it exhibits the same tessellation properties as the quincuncial map projection for the Earth, which maps (in a first step) each hemisphere conformally to a square. This tessellation allows various different map layouts, e. g., in order to center a particular region of interest or for personal taste.

The projection implied by the closed Kohonen map cannot be described analytically. It is neither exactly conformal (shape preserving) nor exactly area preserving, but behaves quite well over most of the surface. The projection is described by a very special shape model, which is not made of triangles — as all other shape models of Chury — but of quadrangles. It has the additional property that the vertices of the quadrangles are ordered along the lines of a two-dimensional grid, so that there is a unique relation between any point on the surface of the three-dimensional shape model and a position on the map defined by the grid.