# Astrometry in Gaia DR1

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on behalf of the AGIS team and the rest of DPAC

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# Outline of talk

The standard astrometric model

➔ kinematic and astrometric parameters

Use of priors in Gaia DR1

➔ TGAS and the secondary solution

Overview of the astrometric content of Gaia DR1

➔ parameters, uncertainties, excess noise

Limitations of Gaia DR1

➔ known or suspected biases

What can be expected from Gaia DR2?

# The standard astrometric model for “stars”

In the standard astrometric model the Gaia source is assumed

- to be a point source (more precisely: have a well-defined photocentre), and
- to move through space at constant velocity relative to the Solar System Barycentre

This is probably a good approximation for >80% of unresolved Gaia sources beyond the Solar System (and for many resolved sources)

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In Gaia DR1 the standard astrometric model is used for ***all*** non-solar system objects (stars, binaries, AGNs, ...)

Deviations are indicated by the ***astrometric excess noise*** (explained later)

(Future DRs will use more specialised models for some sources - not discussed here)

# Kinematic and astrometric parameters

Point source ⇒ well-defined barycentric position vector $\boldsymbol{r}(t)$

Uniform velocity ⇒ $\boldsymbol{r}(t) = \boldsymbol{r}_0 + (t - t_0)\boldsymbol{v}$

**Kinematic model:**

- reference time $t_0$ and six kinematic parameters $x_0$, $y_0$, $z_0$, $v_x$, $v_y$, $v_z$

**Astrometric model:**

- reference epoch $t_{ep}$ and six astrometric parameters $\alpha$, $\delta$, $\varpi$, $\mu_{\alpha*}$, $\mu_\delta$, $v_r$

The two sets of parameters are ***in principle*** equivalent, but:

$t_0$, $x_0$, $y_0$, $z_0$, $v_x$, $v_y$, $v_z$

$t_{ep}$, $\alpha$, $\delta$, $\varpi$, $\mu_{\alpha*}$, $\mu_\delta$, $v_r$

trivial and always possible - note: $t_{ep} = t_0 + |\boldsymbol{r}_0|c^{-1}$

difficult, and not always possible (e.g. when $\varpi \leq 0$)

# Why use astrometric parameters?

Observations of Solar System can be modelled directly in barycentric coordinates $\boldsymbol{r}(t)$

For stars and more distant objects the astrometric parameters are preferred:

- they can always be fitted to astrometric observations
- resulting errors are approximately Gaussian
- they work even for sources at “infinite” distance

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**Example**

Astrometric parameters in Gaia DR1 for the quasar 3C273 (HIP 60936):

$$t_{\rm ep} = 2015.0 \text{ (chosen)}$$

| | |
|---|---|
| $\alpha$ = 187.277915798° ± 0.312 mas* | $\mu_{\alpha*}$ = −0.384 ± 0.443 mas/yr |
| $\delta$ = +2.052388638° ± 0.216 mas | $\mu_{\delta}$ = +0.111 ± 0.288 mas/yr |
| $\varpi$ = −0.140 ± 0.377 mas | $v_r$ = 0 (assumed) |

# Three remarks on the astrometric parameters

1. The sixth parameter, radial velocity $v_r$ (or radial proper motion $\mu_r = \varpi v_r / A$), is ignored in Gaia DR1 (assumed = 0)
   - important for a small number of nearby, high-velocity stars (not in DR1 anyway)
   - gives a quadratic variation of position (perspective acceleration)

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2. The astrometric parameters describe the instantaneous motion at the specified reference epoch $t_{\rm ep}$ (= 2015.0 for Gaia DR1)
   - especially the position parameters (α, δ) depend on $t_{\rm ep}$ due to proper motion
   - the parameters can be transformed to any desired epoch (see documentation)
   - future releases will use a different reference epoch than 2015.0
   - “epoch” not to be confused with “equinox” (e.g. J2000.0 = ICRS)
   - the “equinox” is always the same: ICRS

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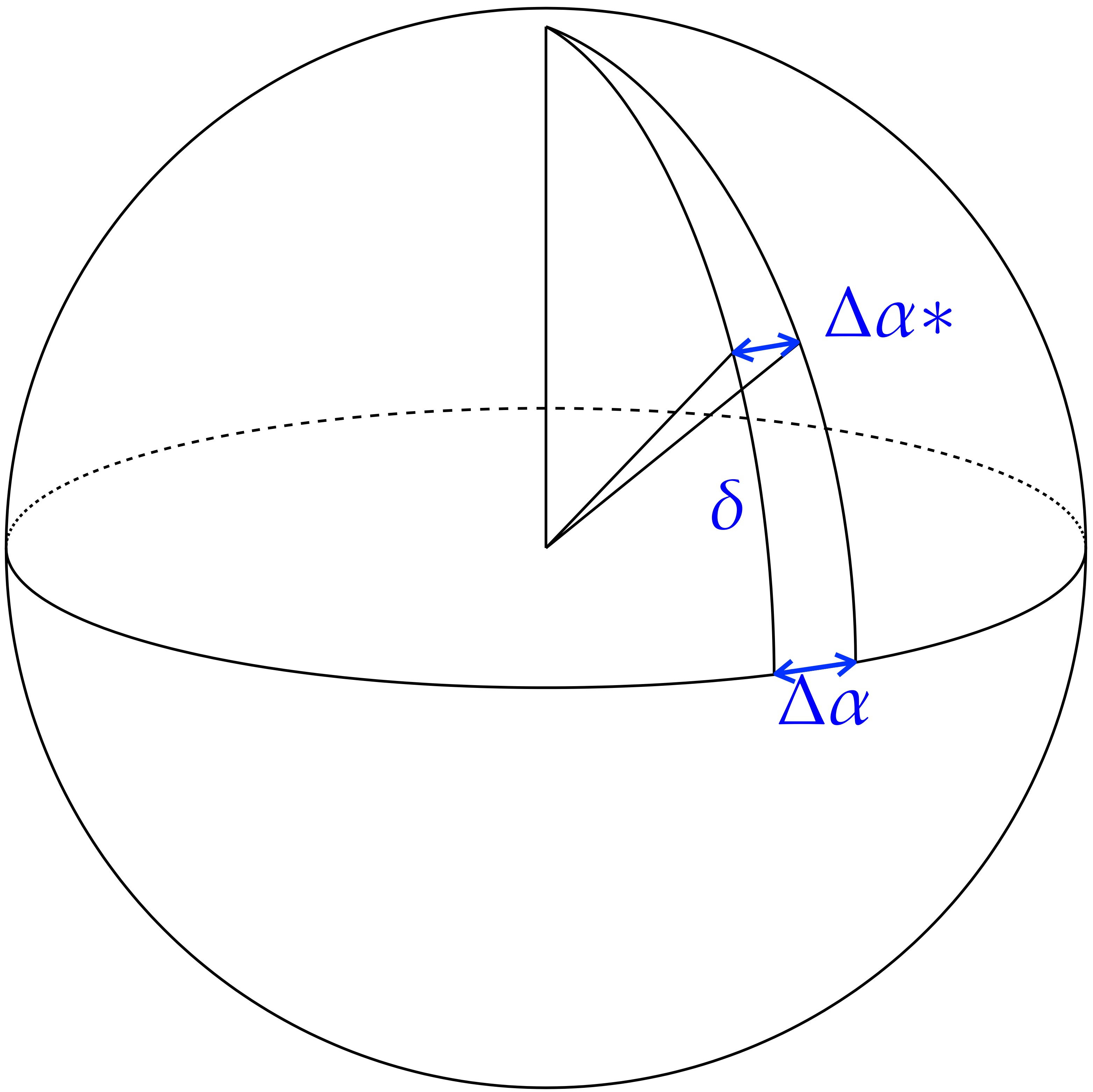
3. The asterisk signifies that a differential quantity in $\alpha$ is a “true arc”:

$$\mu_{\alpha *} = \frac{\mathrm{d}\alpha}{\mathrm{d}t} \cos\delta \,, \quad \sigma_{\alpha *} = \sigma_\alpha \cos\delta \,, \quad \Delta\alpha * = \Delta\alpha \cos\delta$$

# $\Delta\alpha* = \Delta\alpha \cos\delta$



# Use of priors in Gaia DR1

Four different kinds of "prior information" are used in Gaia DR1:

## For all sources:

*sources have no radial motion* ($v_r = 0$)
➔ this is usually an acceptable approximation (except for nearby high-μ stars)

## For the primary (TGAS) solution:

*positions at epoch 1991.25 are known from Hipparcos or Tycho-2*
➔ provides useful proper motions and parallaxes with only ~1 year of data

## For the secondary solution:

*parallaxes and proper motions are small for most stars ("Galactic prior")*
➔ gives positions at 2015.0 with realistic uncertainties

## For the auxiliary quasar solution:

*quasars have negligible proper motion*
➔ accurate quasar positions for alignment with the VLBI reference frame (ICRF)

# Number of sources and parameters in Gaia DR1

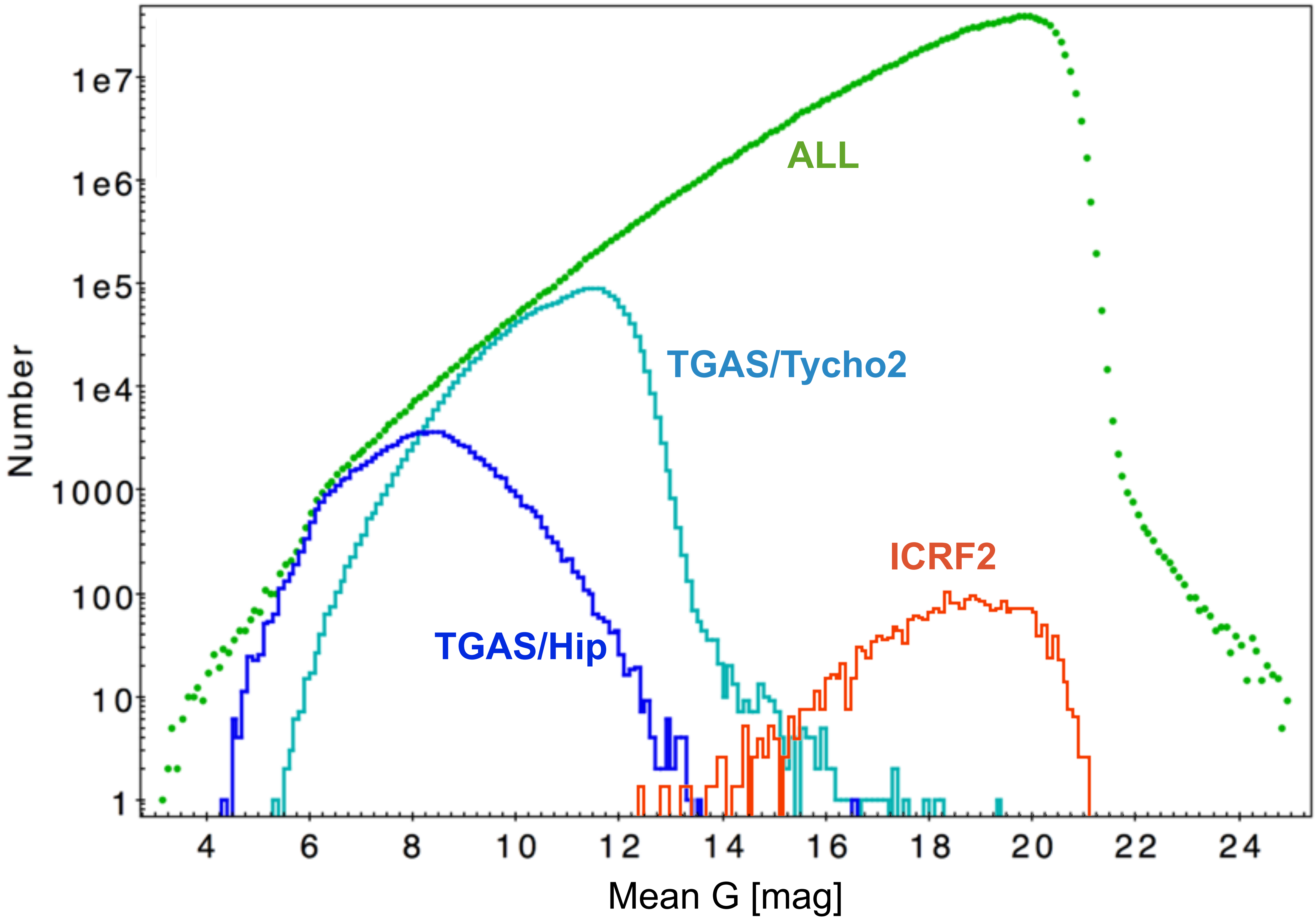
| Solution | No. of sources | Param. | Prior used |
|---|---|---|---|
| Primary (TGAS) sources | 2 057 050 | 5 | positions at 1991.25 |
| - of which Hipparcos | 93 635 | 5 | - Hipparcos positions |
| - of which Tycho-2 (excl Hipp) | 1 963 415 | 5 | - Tycho-2 positions |
| Secondary sources | 1 140 622 719 | 2 | $\varpi$, $\mu_{\alpha*}$, $\mu_\delta$ = 0 ± few mas(/yr) |
| ICRF sources (*) | 2 191 | 2 | $\mu_{\alpha*}$, $\mu_\delta$ = 0 ± 0.01 mas/yr |
| **Total** | **1 142 679 880** | | |

(*) 2080 of the ICRF sources are also secondary sources (with slightly different positions)

**References:**


Michalik et al. 2015, A&A 574, A115 (TGAS)
Michalik et al. 2015, A&A 583, A68 (secondary solution)
Michalik & Lindegren 2016, A&A 586, A26; Mignard et al. 2016, arXiv:1609.07255 (ICRF)
Lindegren et al. 2016, arXiv:1609.04303 (Gaia DR1 astrometry in general)

# Magnitude distributions of Gaia DR1



# Primary (TGAS) sources

2.06 M sources, mainly $G < 11.5$

- this is about 80% of the Hipparcos & Tycho-2 catalogues

Missing sources:

- brights stars ($G < 6$)
- high-proper motion stars ($\mu > 3.5$ ″/yr)
- some 20% of Hip + Tycho-2 with too few observations (quasi-random but with large variations over the sky)

Median **position** uncertainty: 0.23 mas at 2015.0

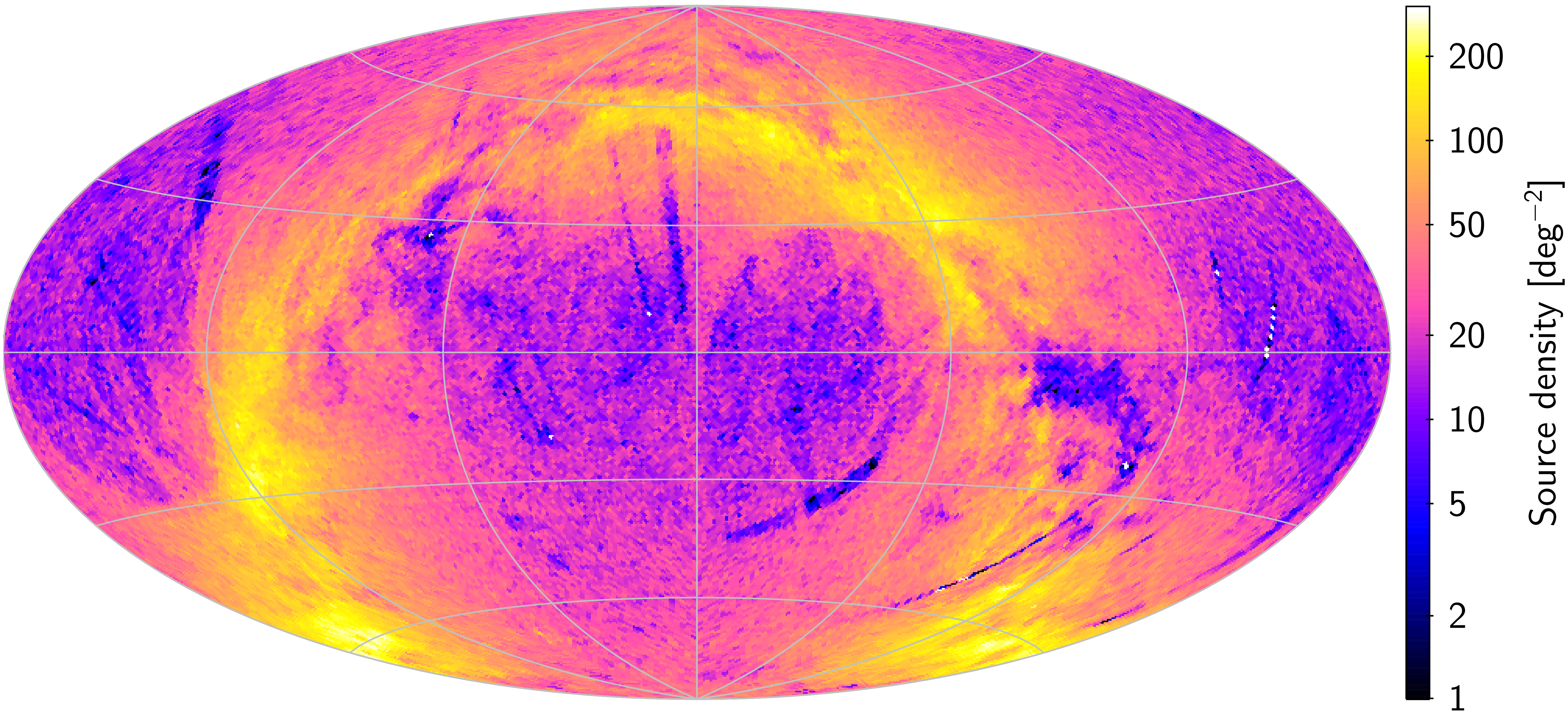
Median **parallax** uncertainty: 0.32 mas

Median **proper motion** uncertainty:

- 0.07 mas/yr (Hipparcos subset)
- 1.2 mas/yr (Tycho-2 subset)

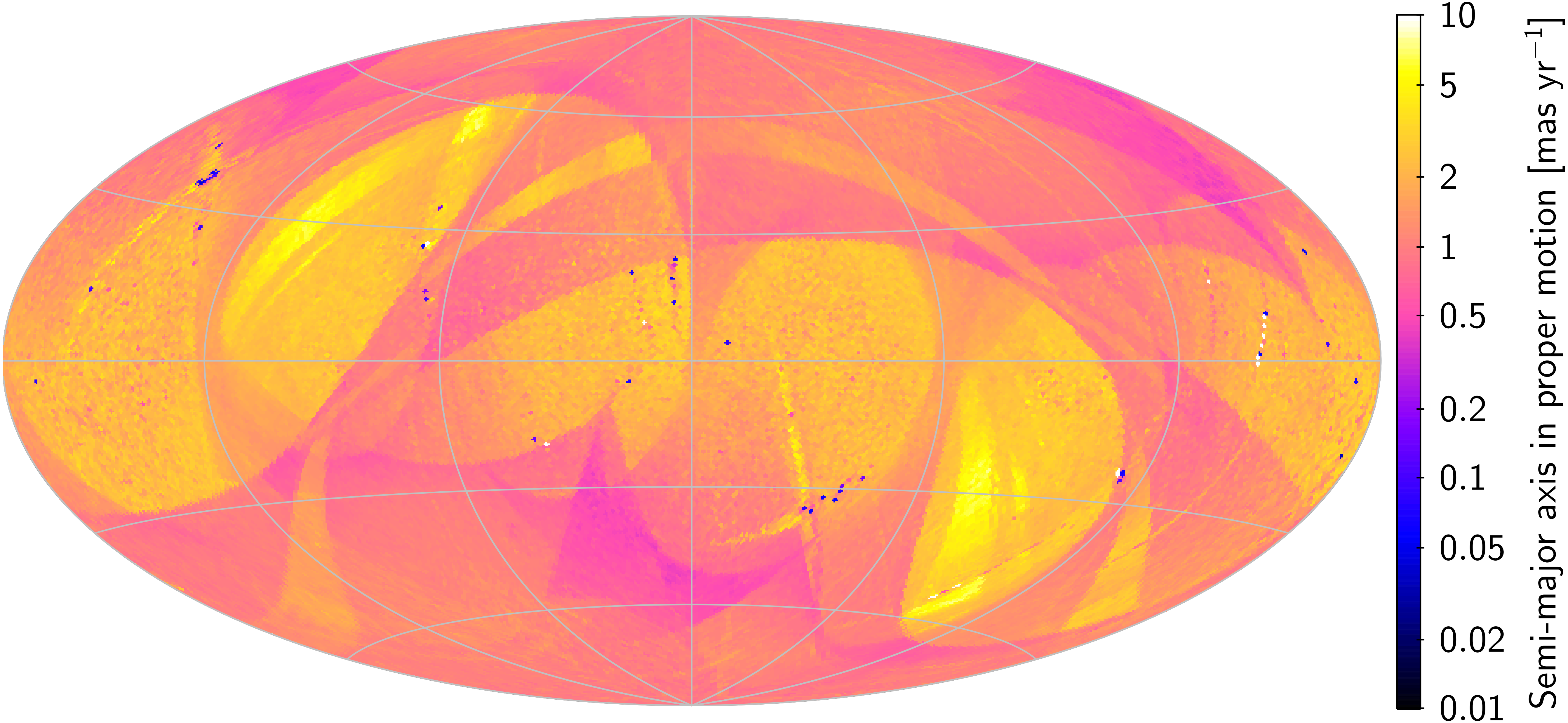
Note difference!

# TGAS: Sky coverage (equatorial map)




Mean density per pixel (~1 deg$^2$)

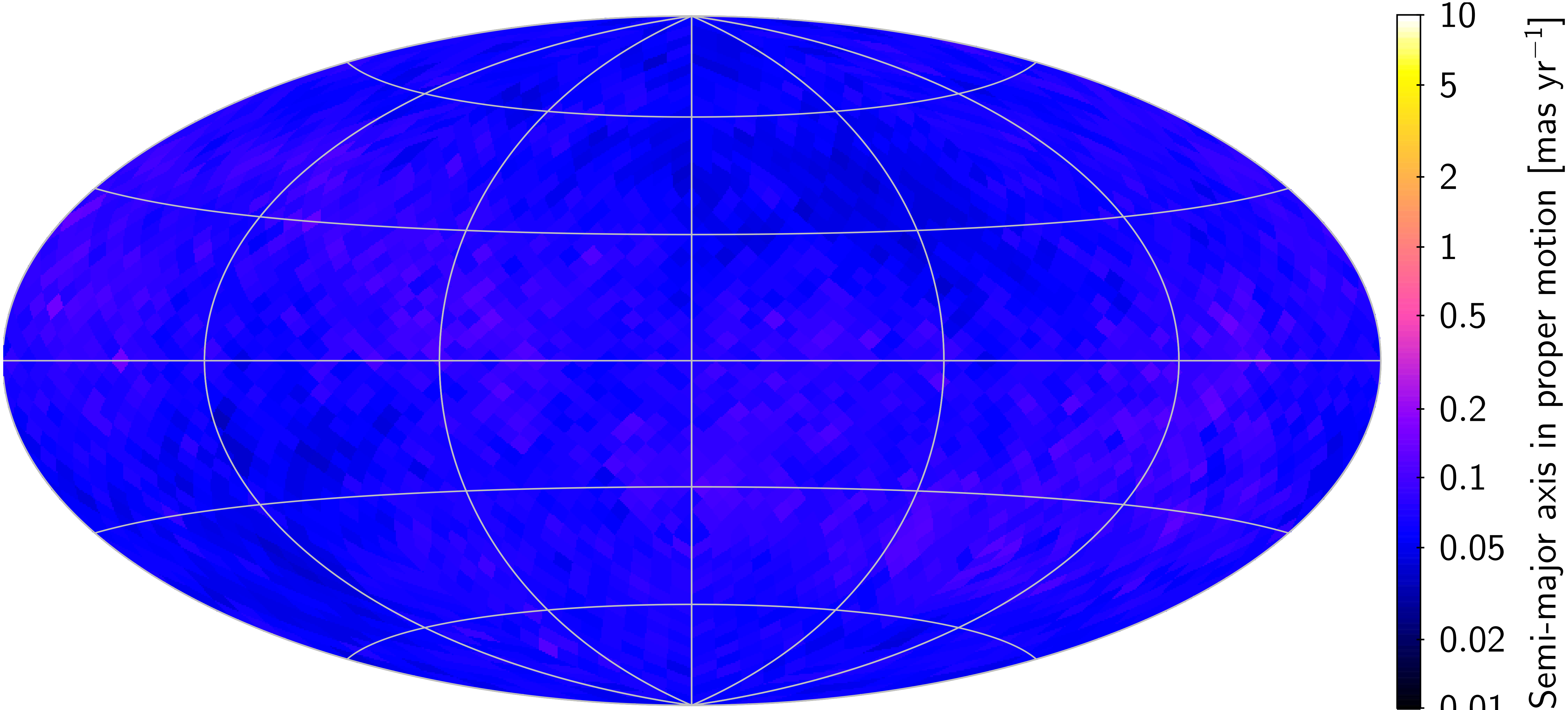
# TGAS: Standard uncertainty in proper motion (semi-major axis of error ellipse) - All sources




Median uncertainty per pixel (~1 deg$^2$)

Overall median = 1.32 mas/yr

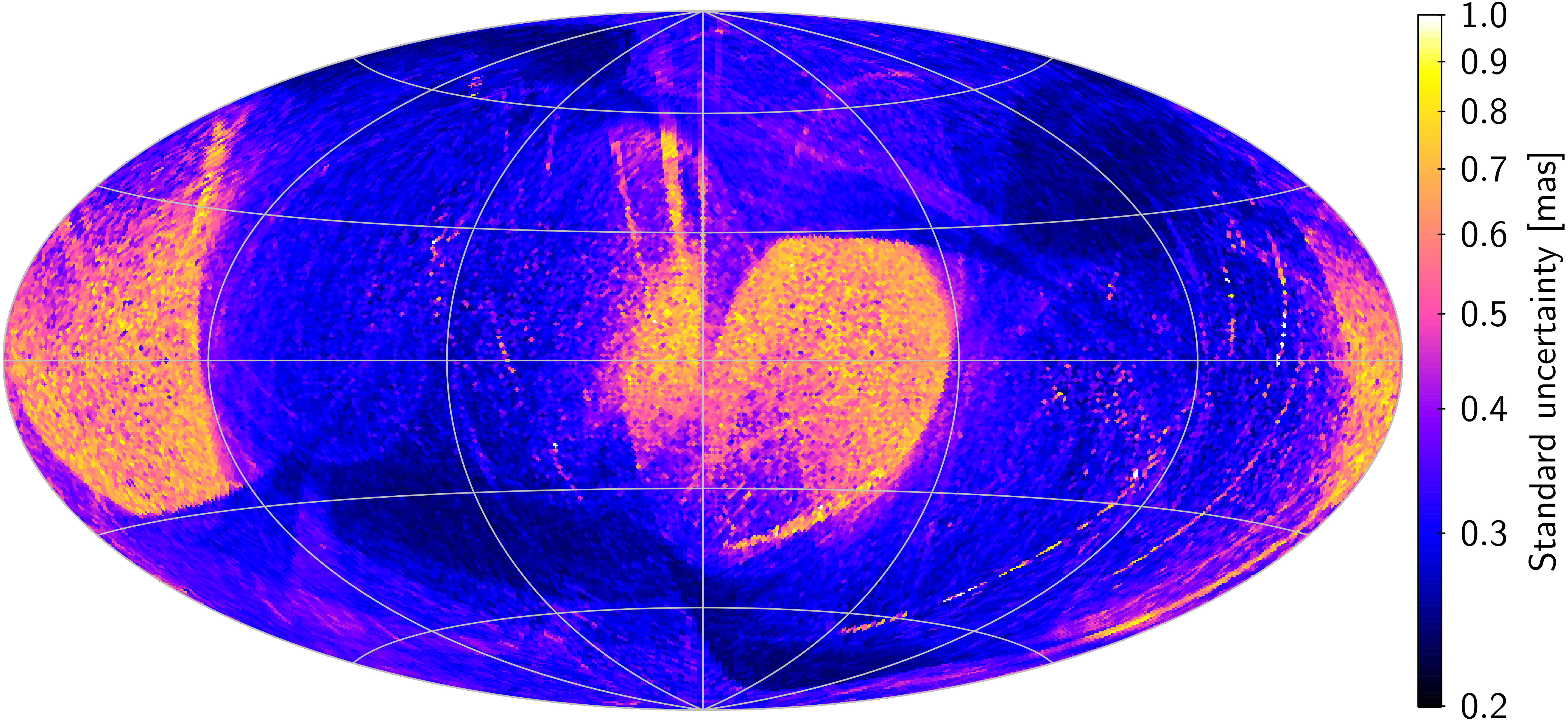
# TGAS: Standard uncertainty in proper motion (semi-major axis of error ellipse) - Hipparcos




Median uncertainty per pixel (~16 deg$^2$)

Overall median = 0.07 mas/yr

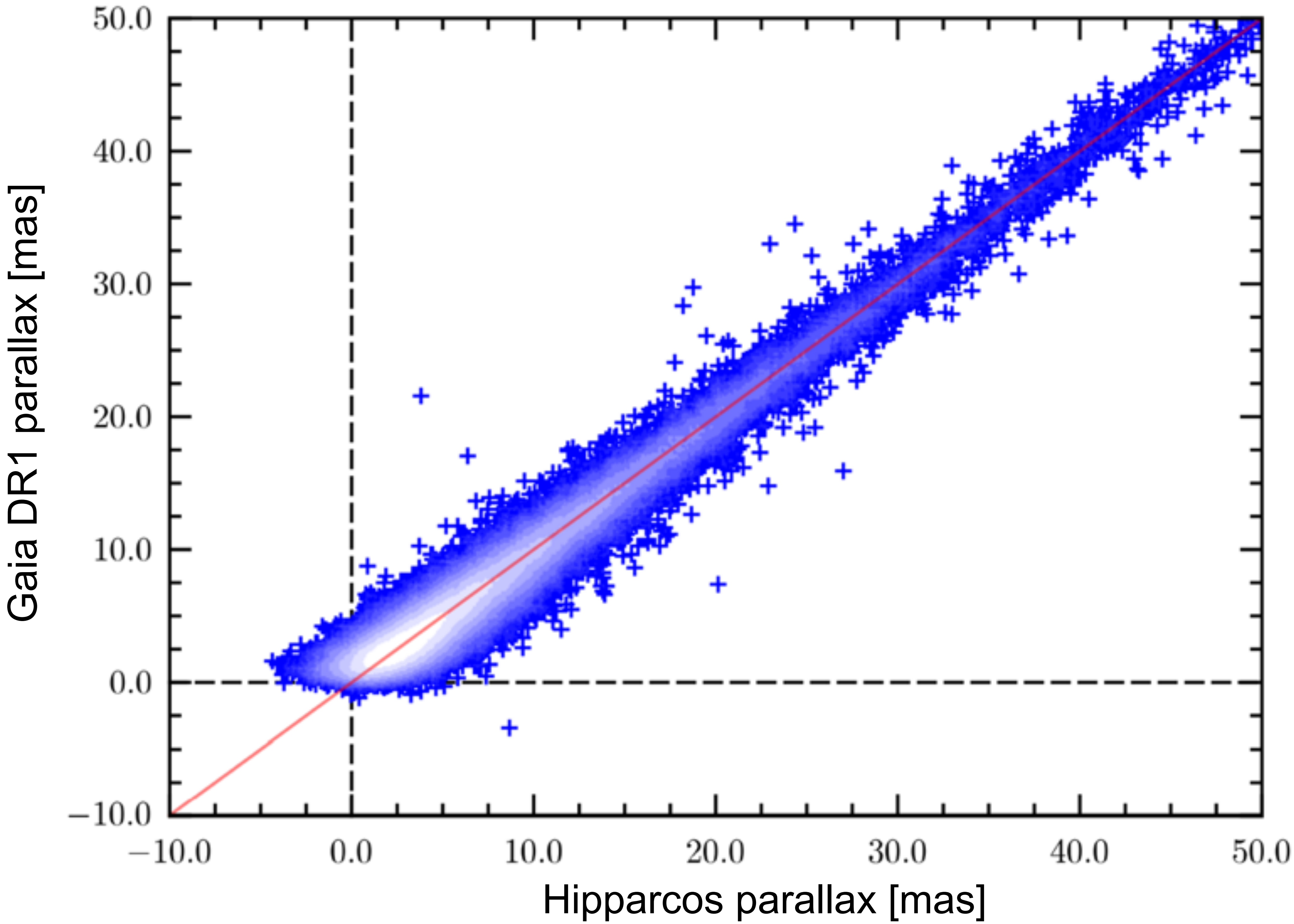
# TGAS: Standard uncertainty in parallax




Median uncertainty per pixel (~1 deg$^2$)

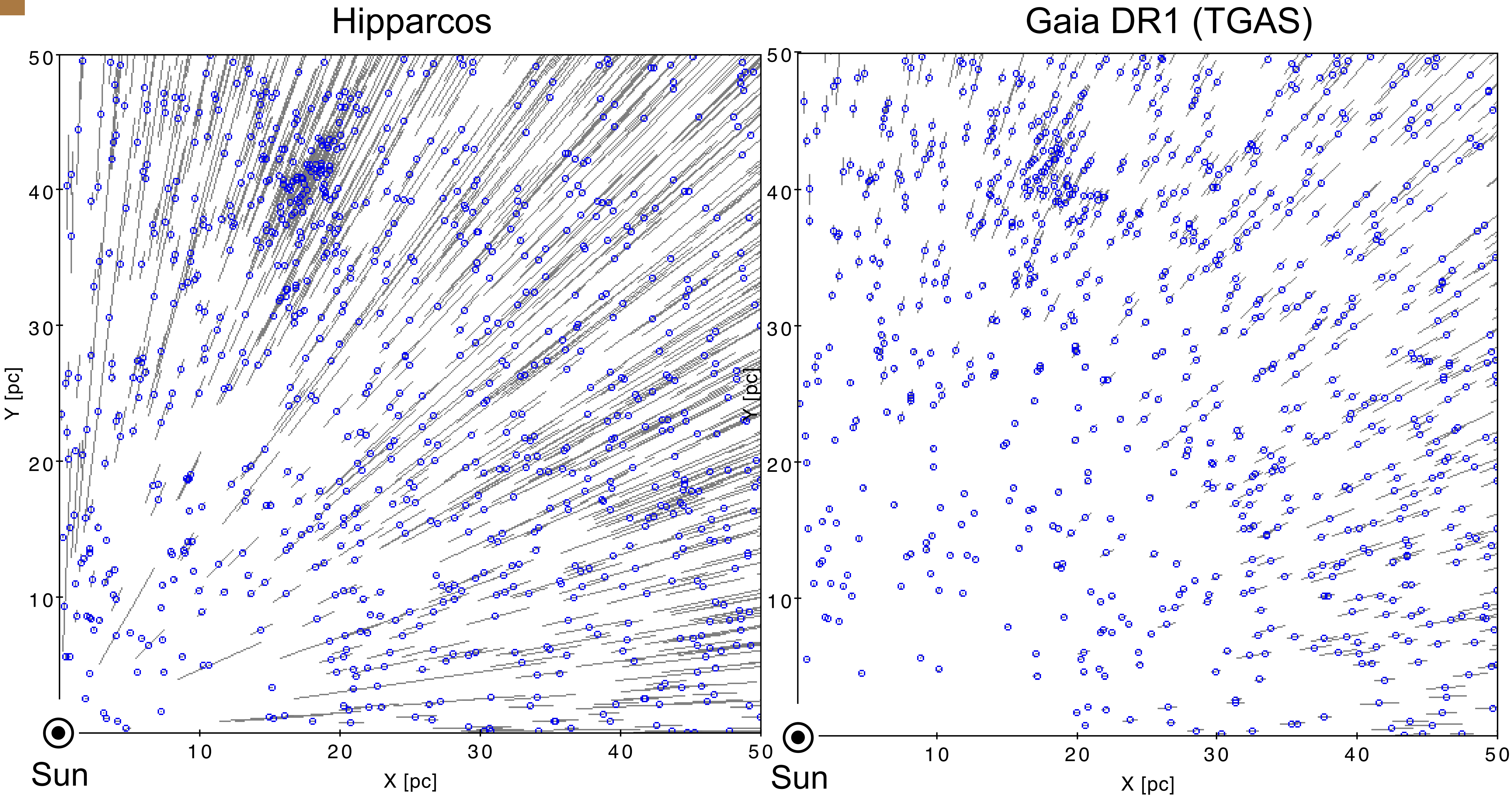
Overall median = 0.32 mas

# Can TGAS parallaxes be trusted?

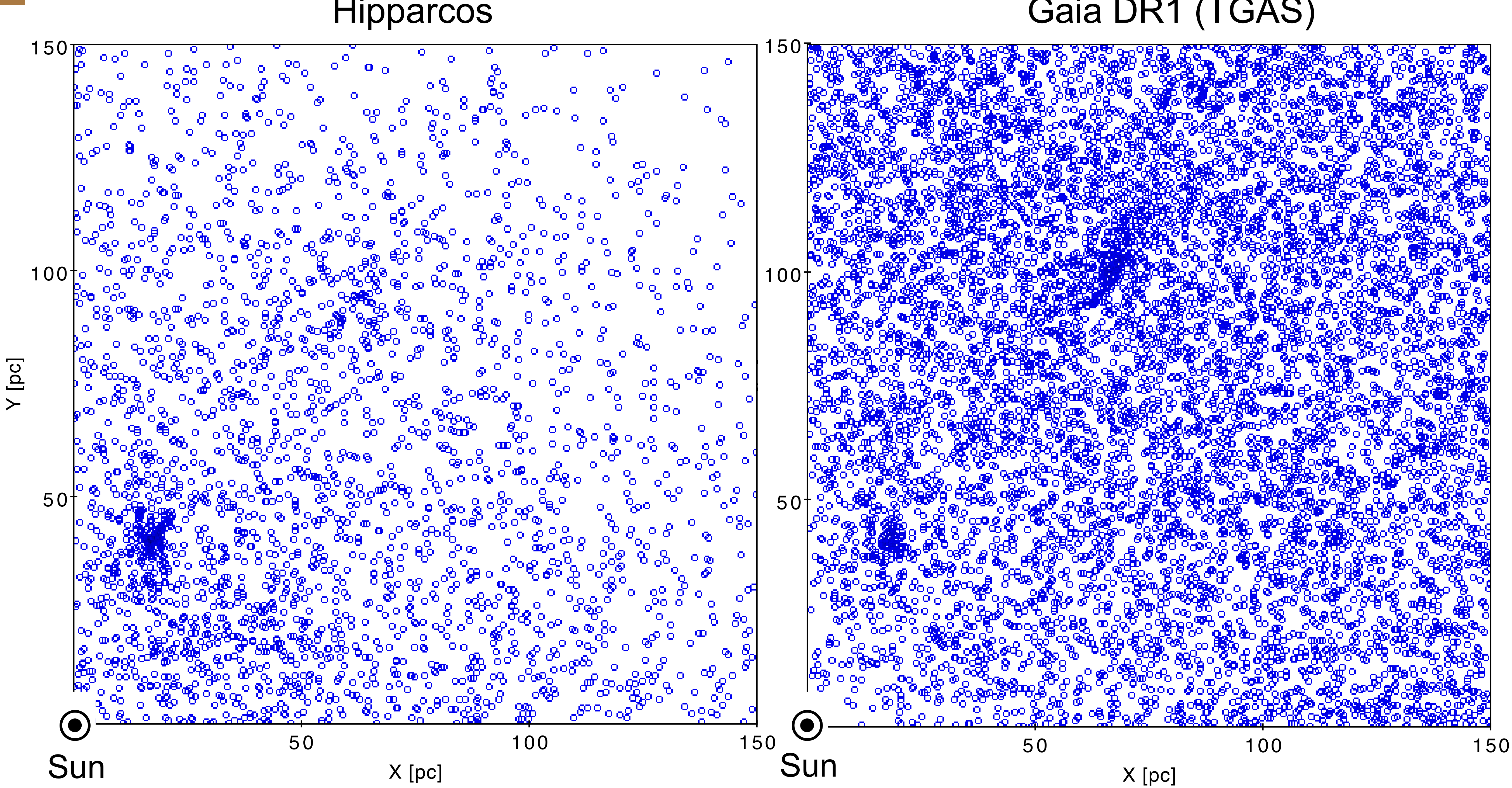



- TGAS and Hipparcos parallaxes are independent!
- Comparison confirms global quality of Hipparcos and Gaia
- Analysis provides realistic error estimates
- Realistic errors are published in Gaia DR1

# Improved distances to nearby stars



# More stars within parallax horizon ($\varpi/\sigma_\varpi > 5$)



# Astrometric quantities in Gaia DR1

**Important:**

- source_id
- ref_epoch (always = 2015.0 in DR1)
- ra, dec
- ra_error, dec_error
- astrometric_excess_noise

- hip, tycho2_id
- parallax, pmra, pmdec
- parallax_error, pmra_error, pmdec_error

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# Astrometric excess noise: Background

The astrometric solution can be formulated as a chi-square minimisation problem

$$\arg\min\ X^2(\boldsymbol{s},\boldsymbol{a},\boldsymbol{c}) = \sum_{\text{sources } i}\ \sum_{\text{obs } j\in i} \left(\frac{R_{ij}}{\sigma_{ij}}\right)^2$$

where ***s***, ***a***, ***c*** are the source, attitude, calibration parameters, $R_{ij}$ the residuals of source *i* in observation *j*, and $\sigma_{ij}$ the formal uncertainty of the observation.

If the model is correct, we expect $X^2_{\text{min}} \sim \chi^2_n$, so $X^2_{\text{min}}/n \simeq 1$ where *n* = degrees of freedom.

In practice the model is never correct, at least not for all sources, so typically we find $X^2_{\text{min}}/n \gg 1$, too much weight are given to bad sources, and the uncertainties of ***s***, ***a***, ***c*** are underestimated.

# Astrometric excess noise: Definition

The problem is instead formulated as

$$\arg\min\ X^2(\boldsymbol{s},\boldsymbol{a},\boldsymbol{c}) = \sum_{\text{sources } i}\ \sum_{\text{obs } j\in i} \frac{R_{ij}^2}{\sigma_{ij}^2+\varepsilon_i^2}$$

For every source, the excess source noise $\varepsilon_i$ is set to the smallest value for which

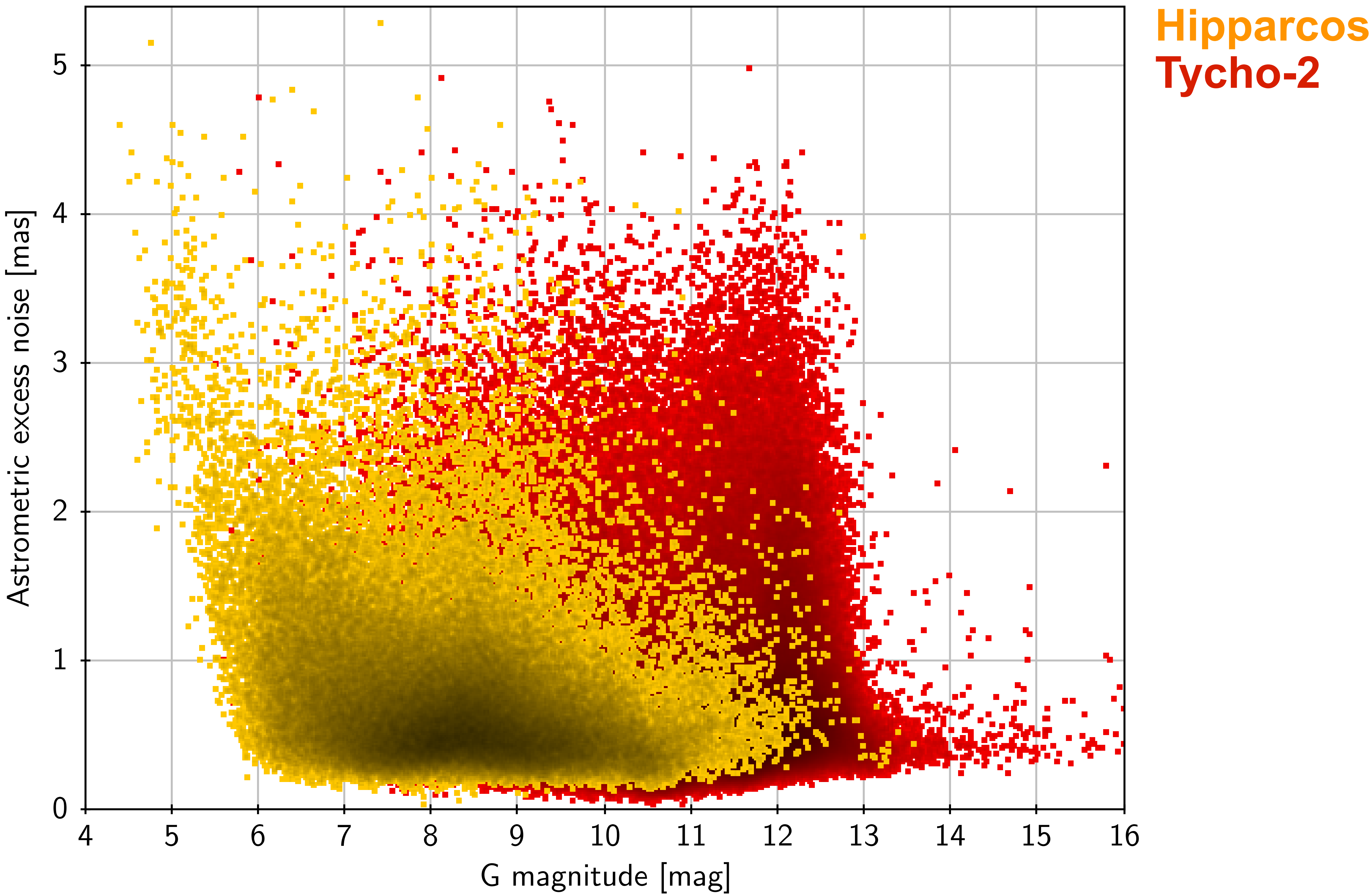
$$\sum_{\text{obs } j\in i} \frac{R_{ij}^2}{\sigma_{ij}^2+\varepsilon_i^2} \leq n_i$$

where $n_i$ is the number of degrees of freedom for source *i*

**Remarks:**

- The excess noise is an angle (in mas)
- Binaries and other badly fitting sources should get large values of $\varepsilon_i$
- Unfortunately, attitude and instrument modelling errors also increase $\varepsilon_i$

# Excess noise versus magnitude (TGAS)



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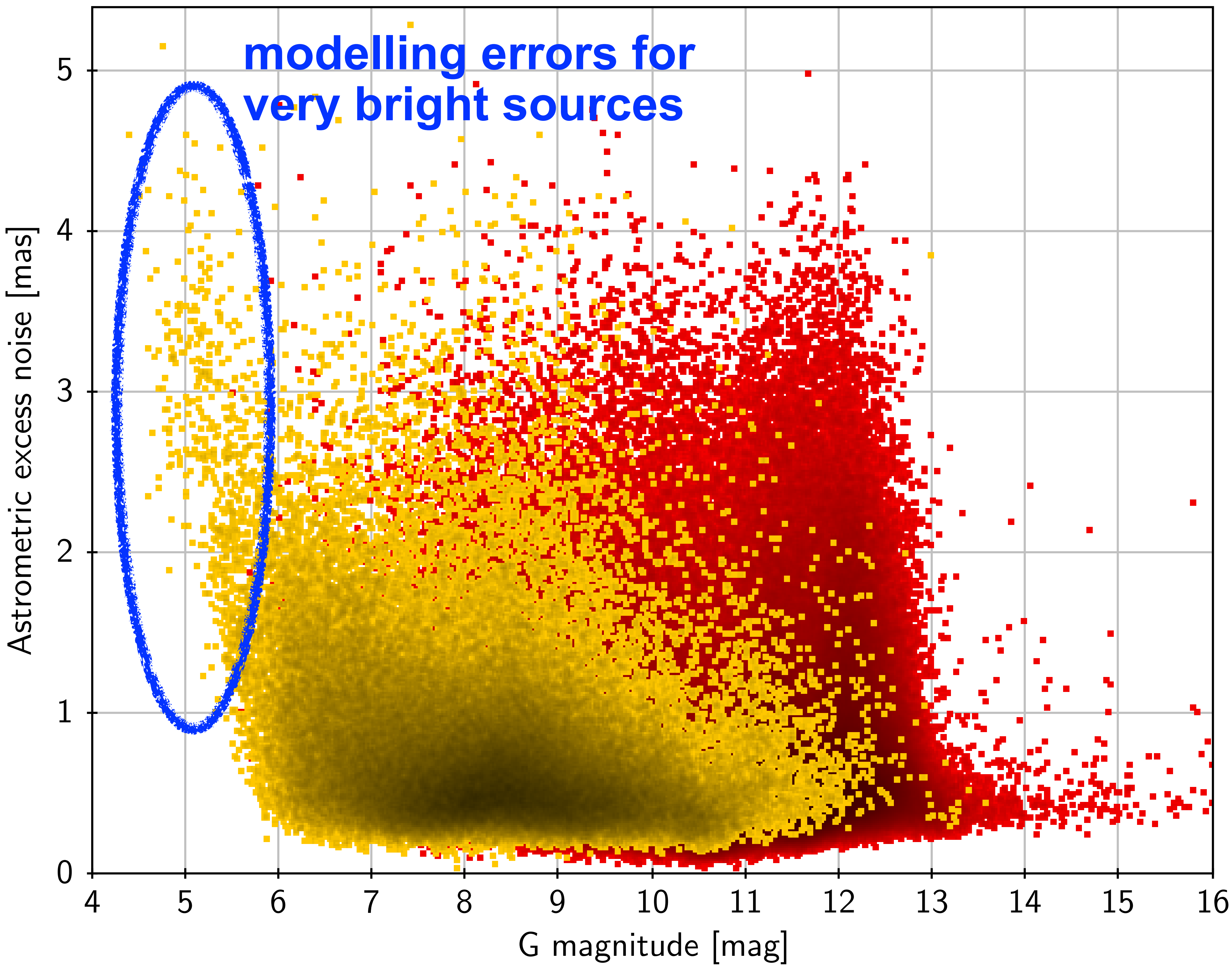
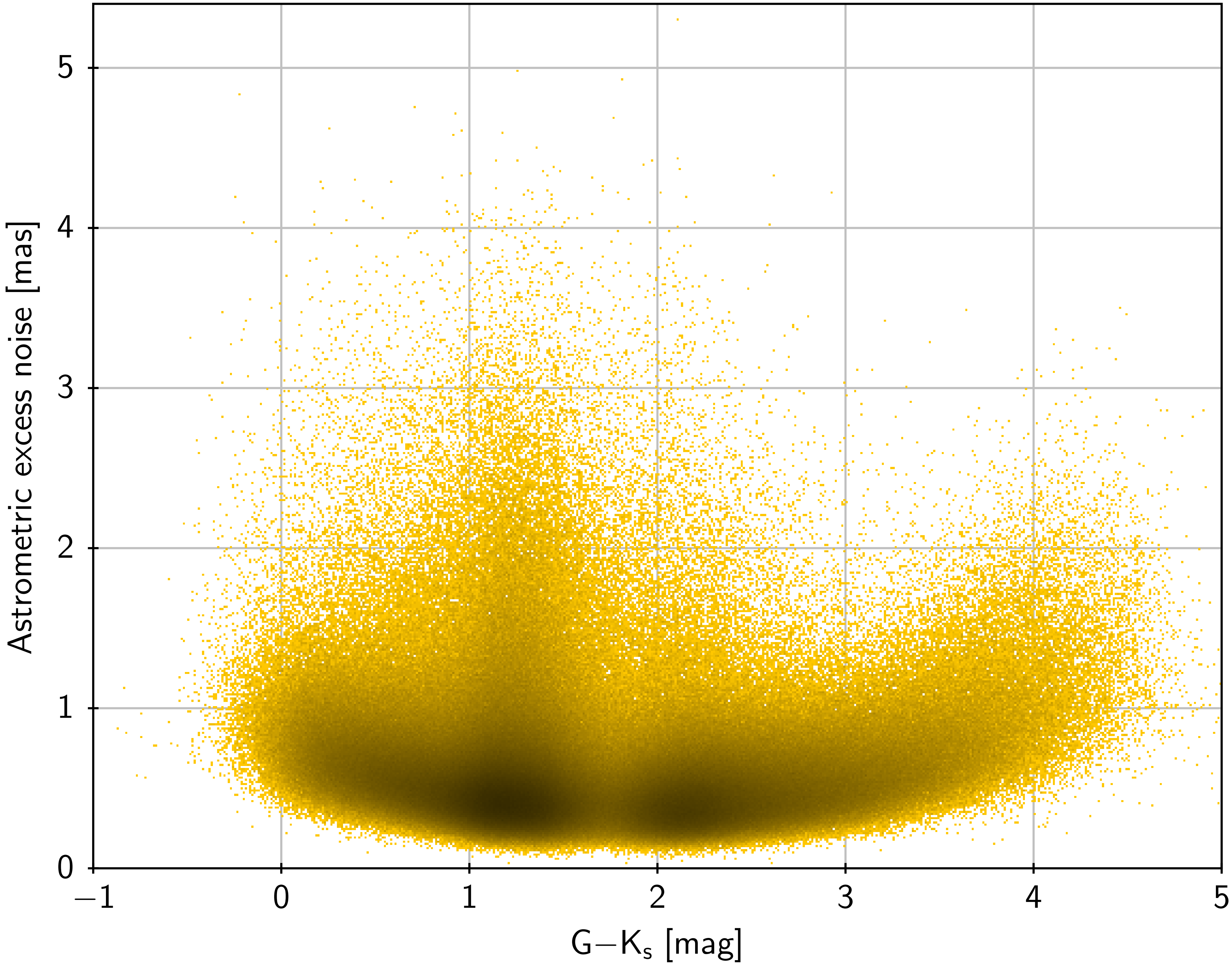



Hipparcos
Tycho-2

# Excess noise versus colour index (TGAS)



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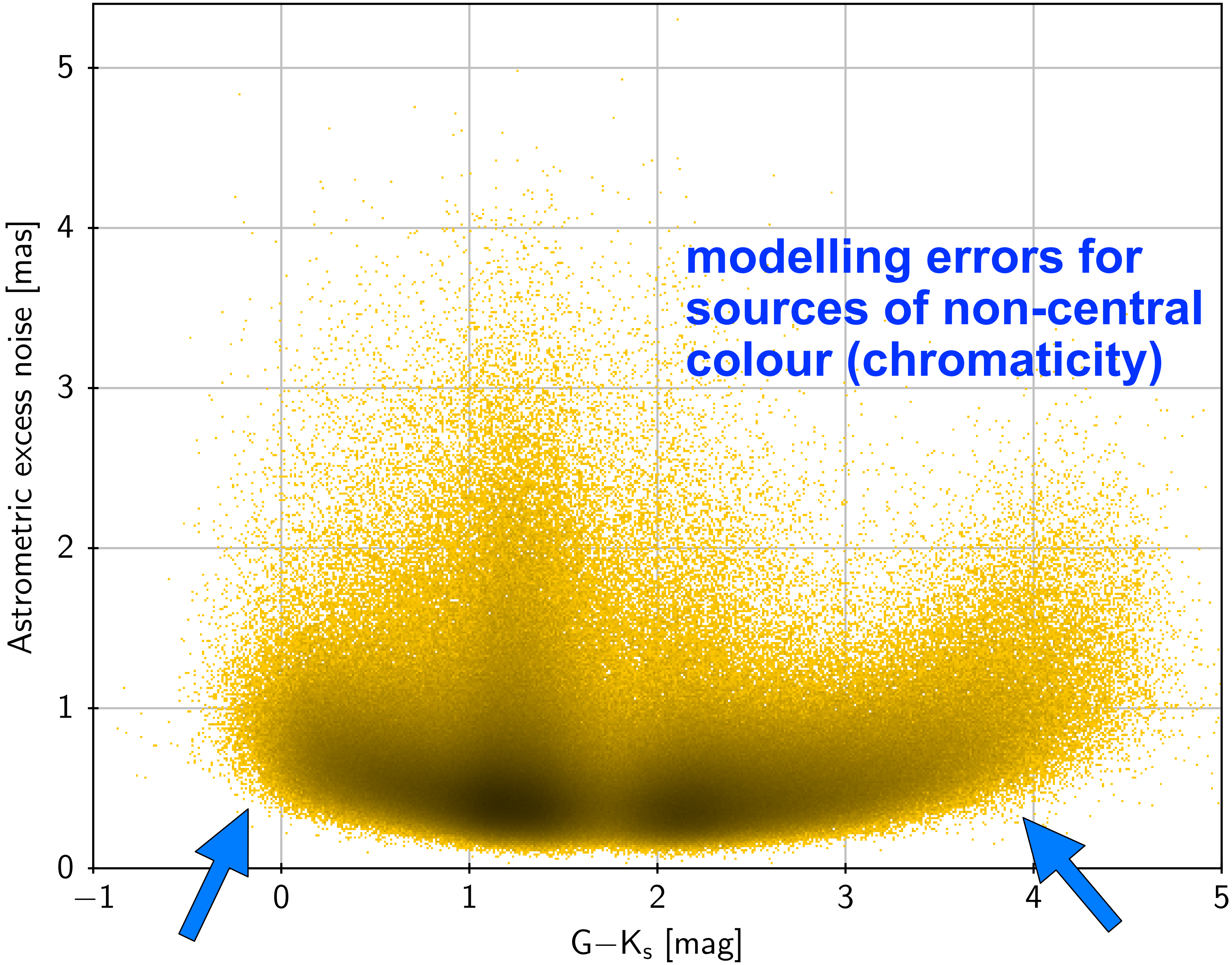
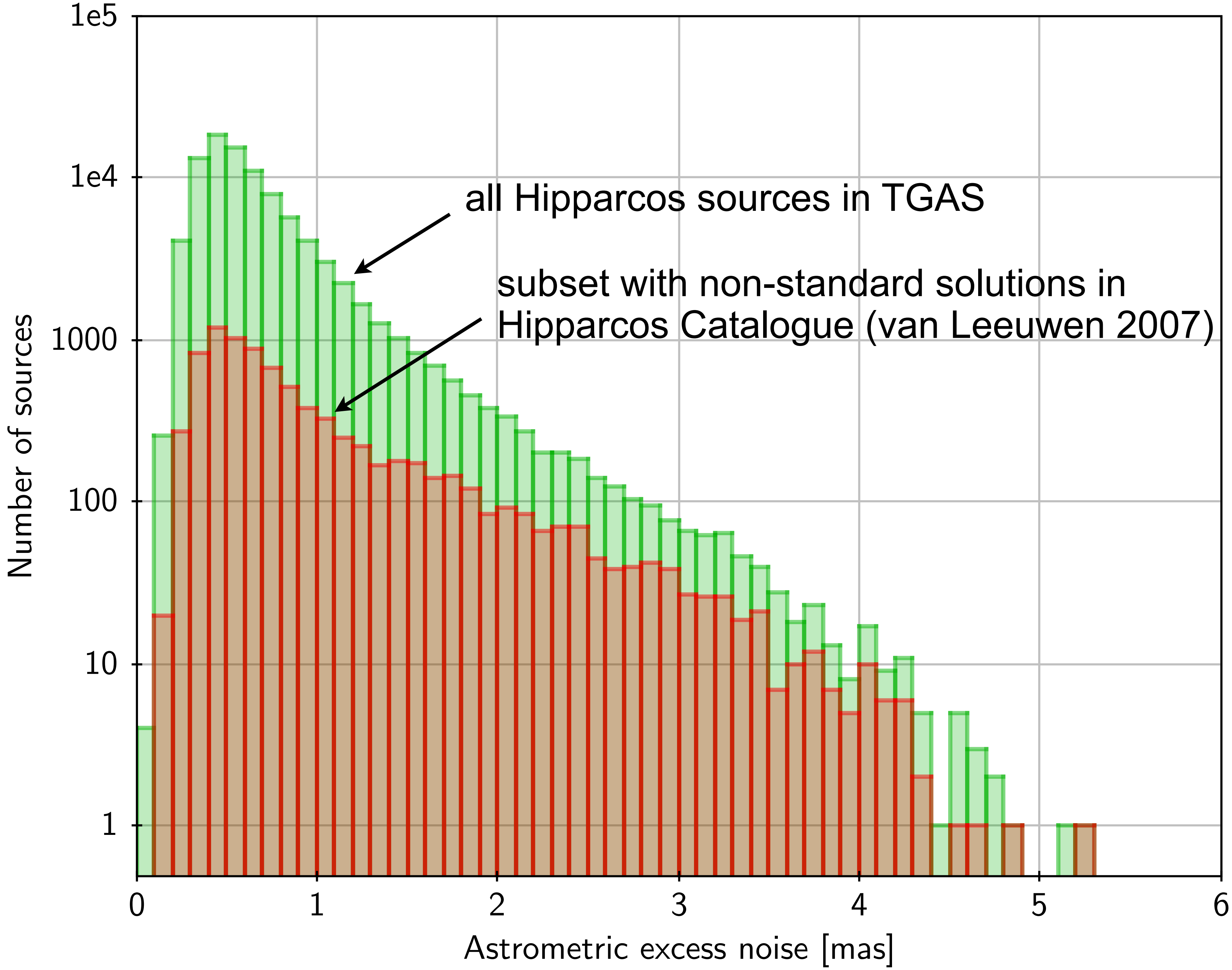
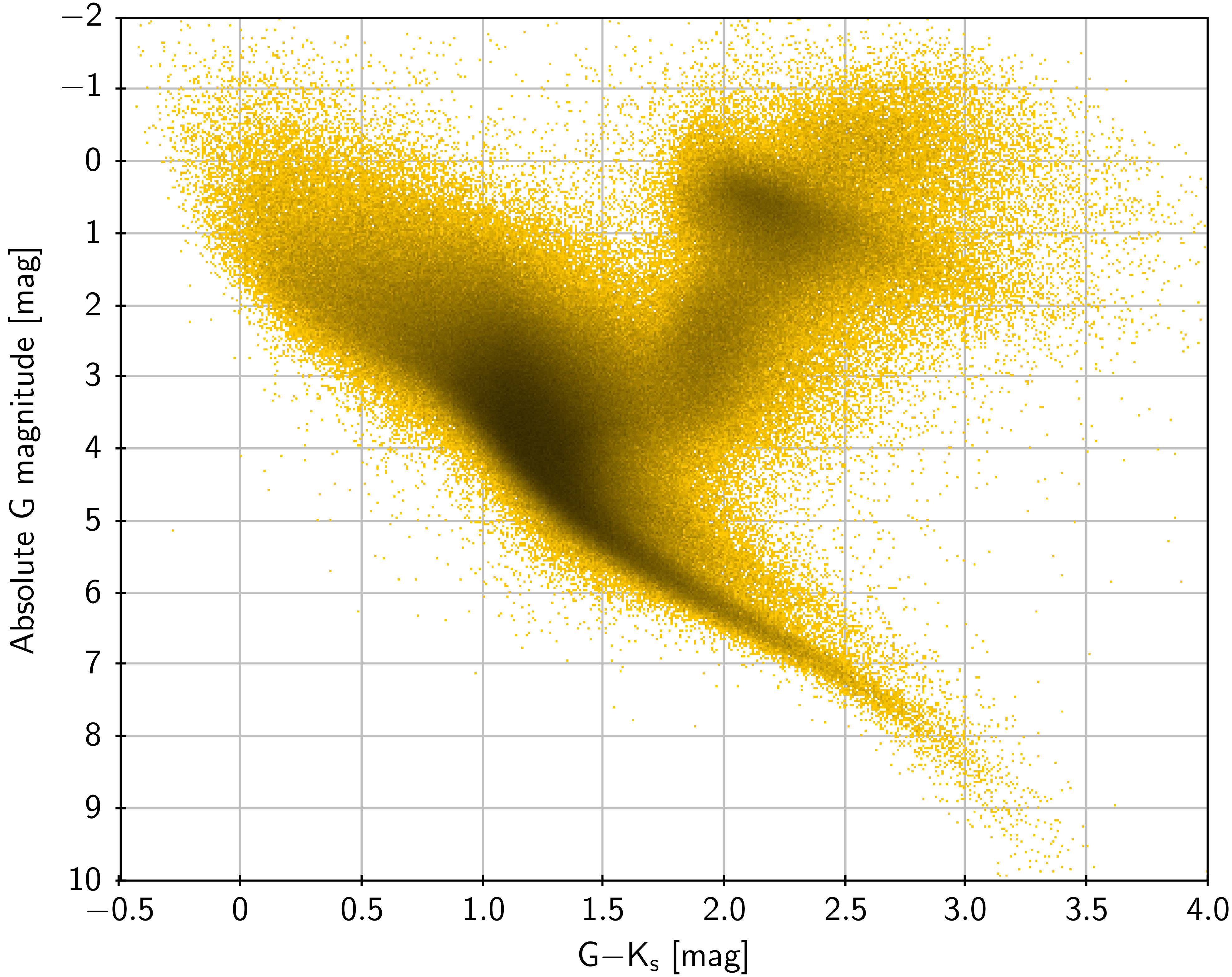


# Excess noise distribution (TGAS/Hip)




The fraction of problematic sources increases with the excess noise

# Binary sequence in HR diagram



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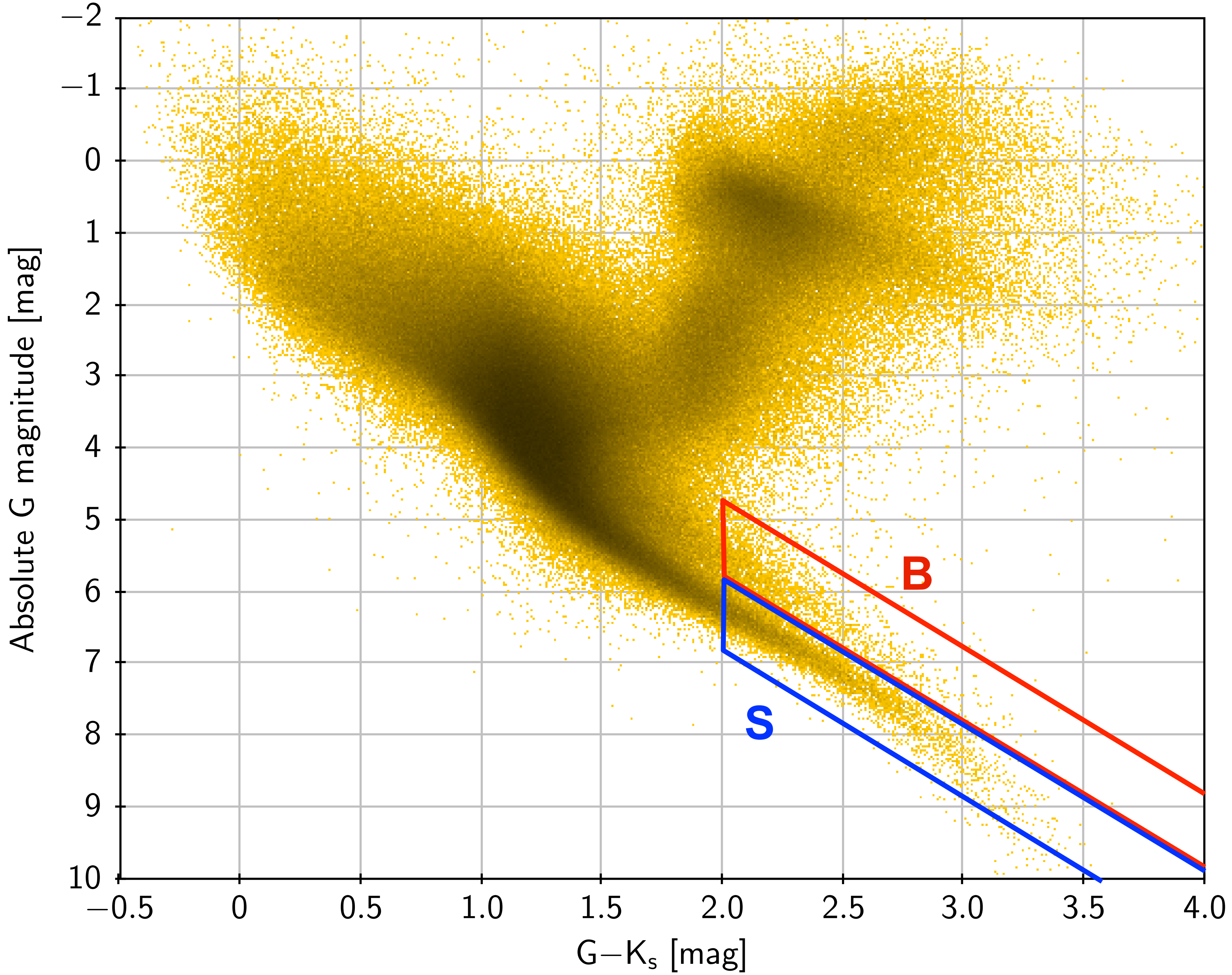
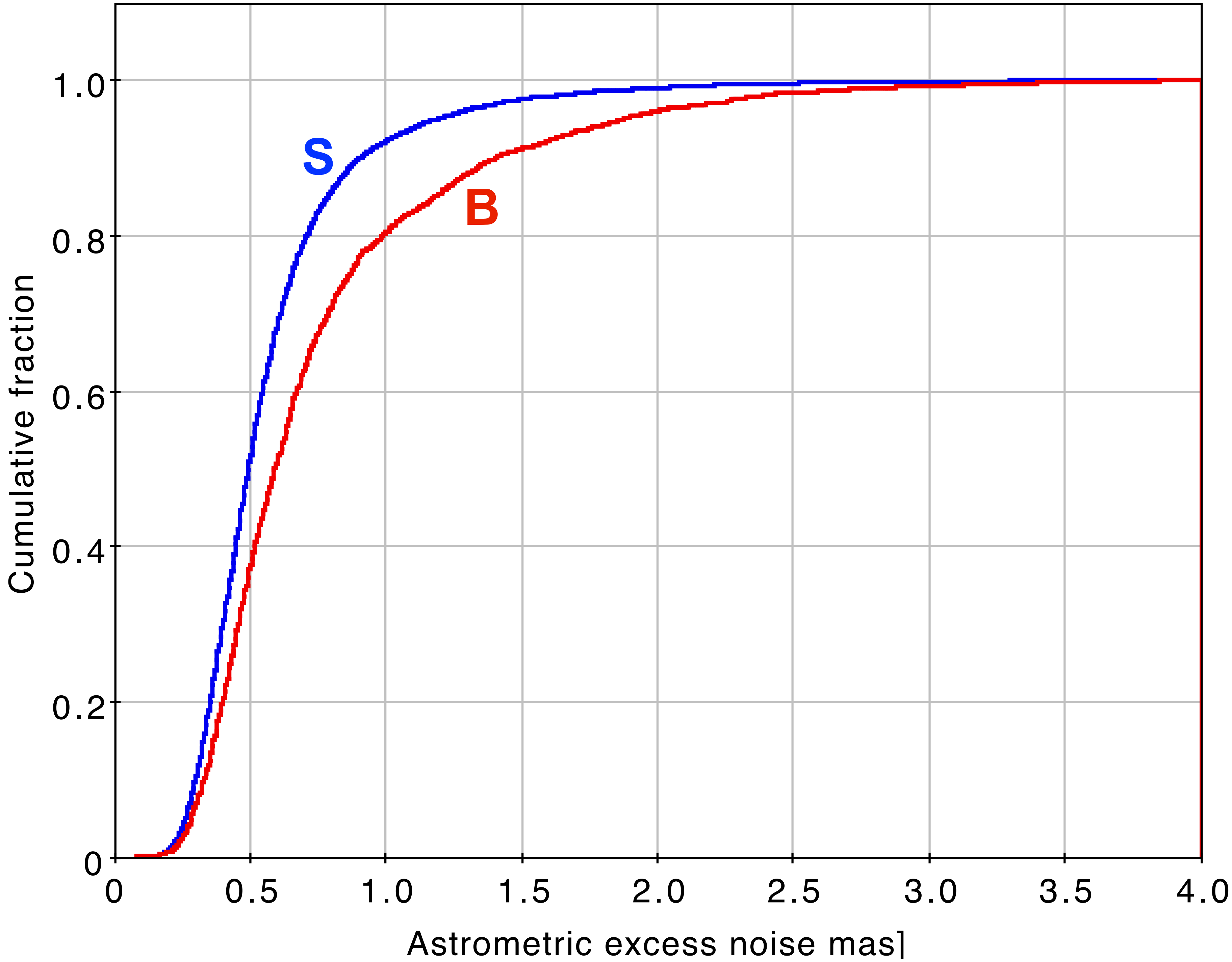


# Distribution of excess noise for sample S and B




Excess noise > 1 mas is twice as common in sample B as in S

# Systematic errors (bias) in Gaia DR1

There are systematic errors in Gaia DR1!

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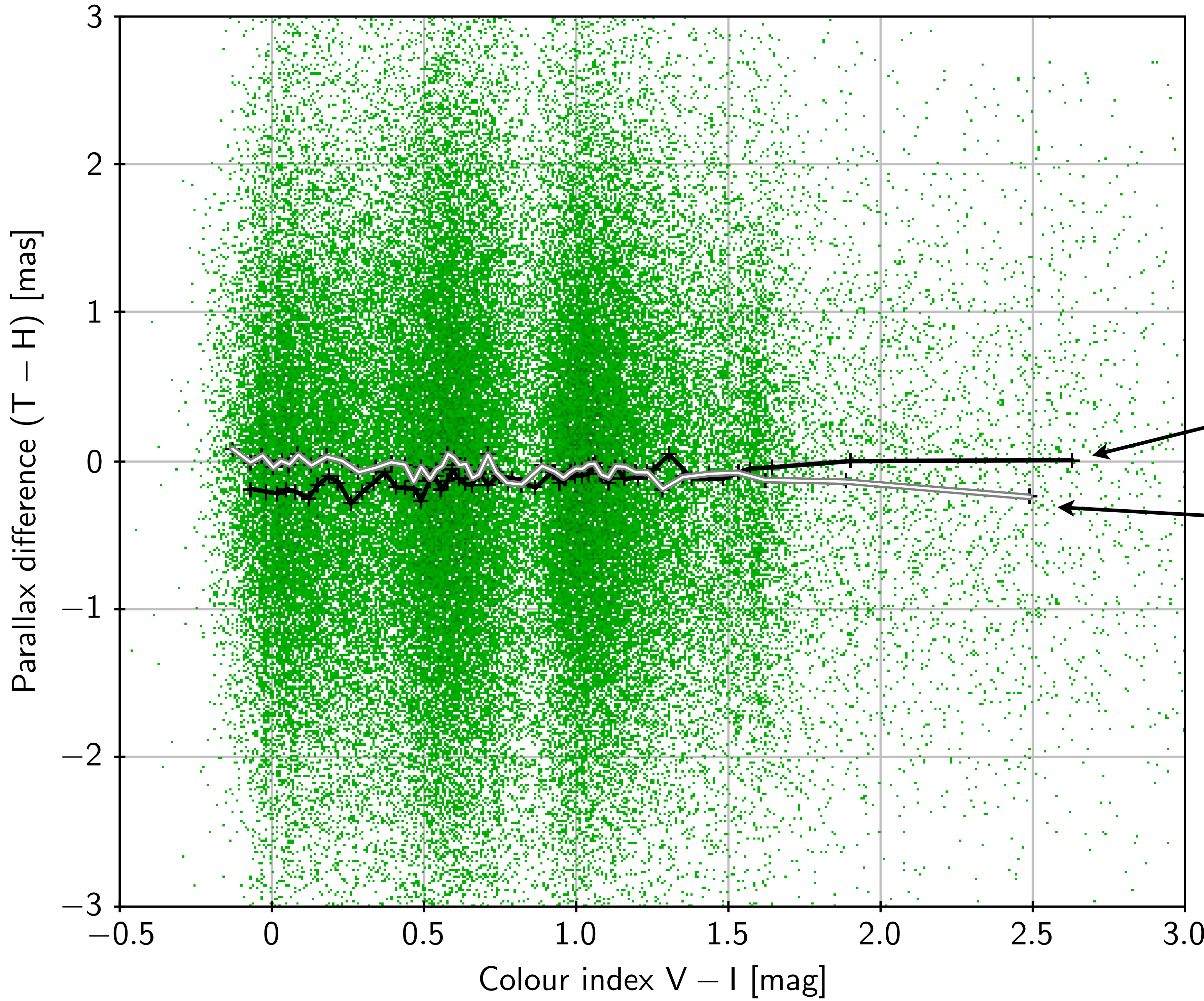
They are complicated (and largely unknown) functions of many things:
position, magnitude, colour, number of observations, prior used, ...

# Systematic errors (bias) in TGAS parallaxes: - Comparison with Hipparcos (FvL 2007)



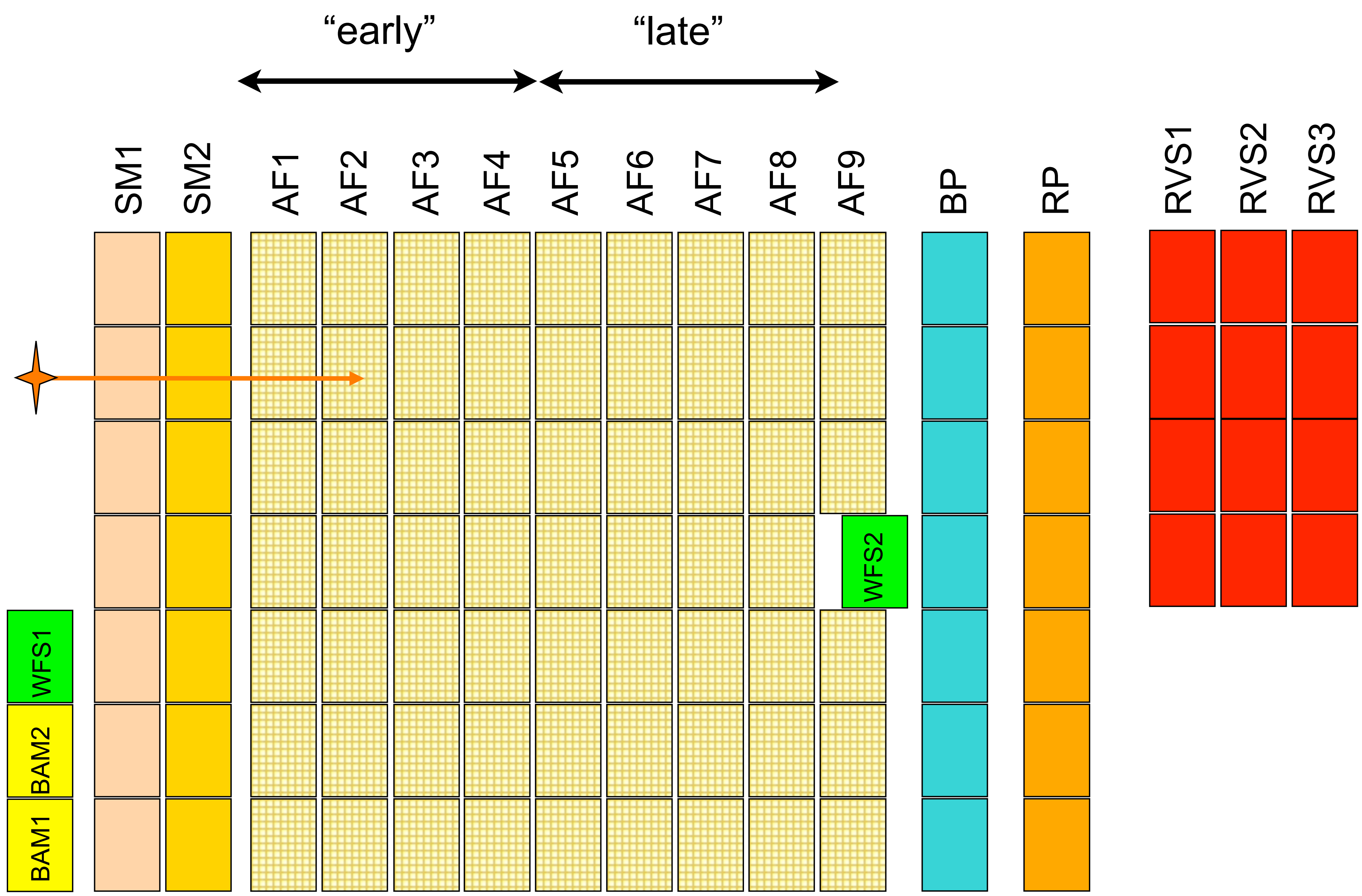

Colour & position dependent systematics on the level ±0.1 mas

β > 0 (ecliptic north)

β < 0 (ecliptic south)

Lines connect median values in 50 colour bins

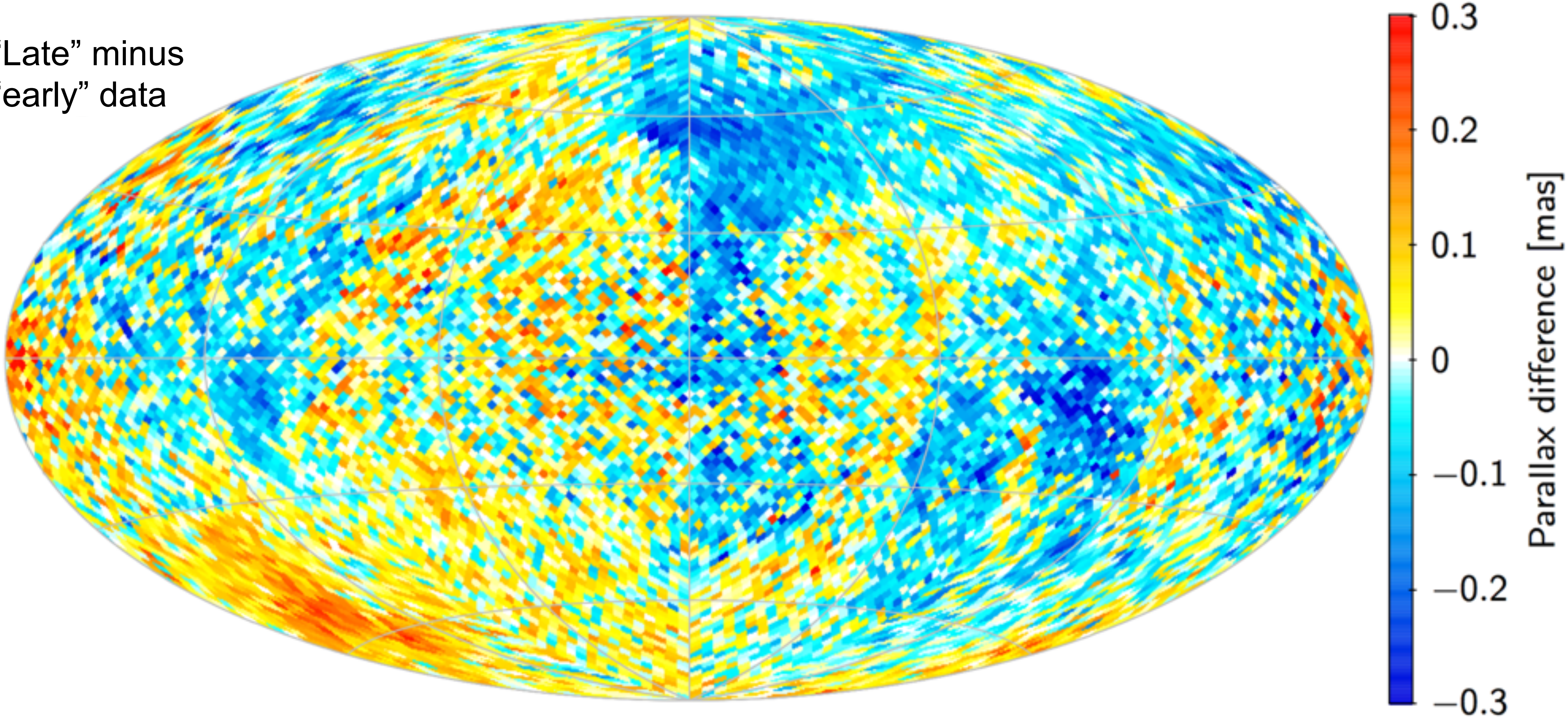
# Split FoV



“early”
“late”
SM1
SM2
AF1
AF2
AF3
AF4
AF5
AF6
AF7
AF8
AF9
BP
RP
RVS1
RVS2
RVS3
WFS2
WFS1
BAM2
BAM1

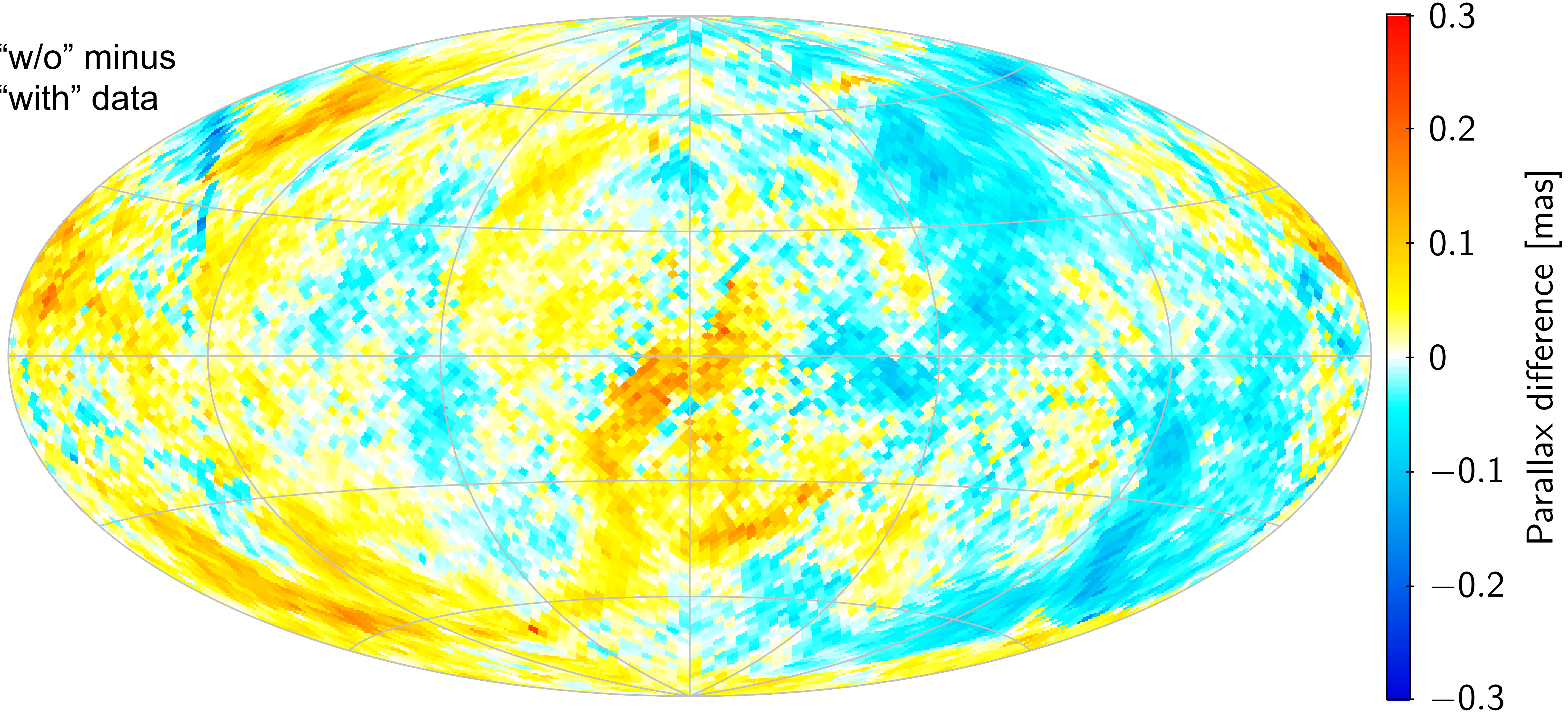
# Systematic errors (bias) in TGAS parallaxes: Comparing solutions from split FoV

“Late” minus
“early” data



# Systematic errors (bias) in TGAS parallaxes: Comparing solutions with and w/o colour terms

"w/o" minus
"with" data



# Systematics in Gaia DR1 parallaxes

Due to known limitations in the astrometric processing

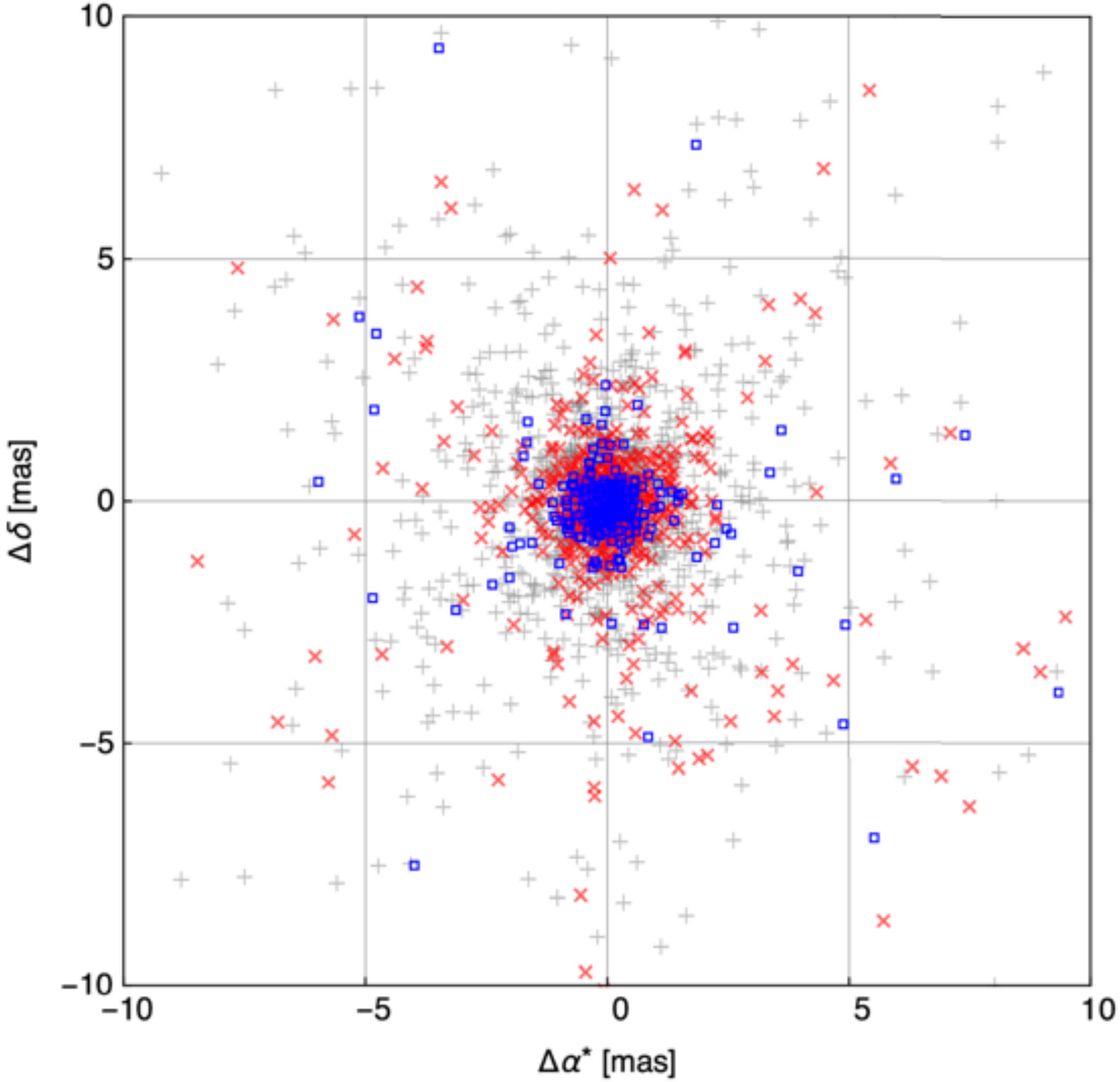
- a global offset of ±0.1 mas may be present
- there are colour dependent, spatially correlated errors of ±0.2 mas
- over large spatial scales, parallax zero point errors reach ±0.3 mas
- in a few small areas even ±1 mas

Parallax uncertainties should be quoted as

$$\varpi \pm \sigma_{\varpi} \text{ (random)} \pm 0.3 \text{ mas (syst.)}$$

Averaging parallaxes e.g. in a cluster does not reduce the systematics!

# Reference frame from observations of quasars



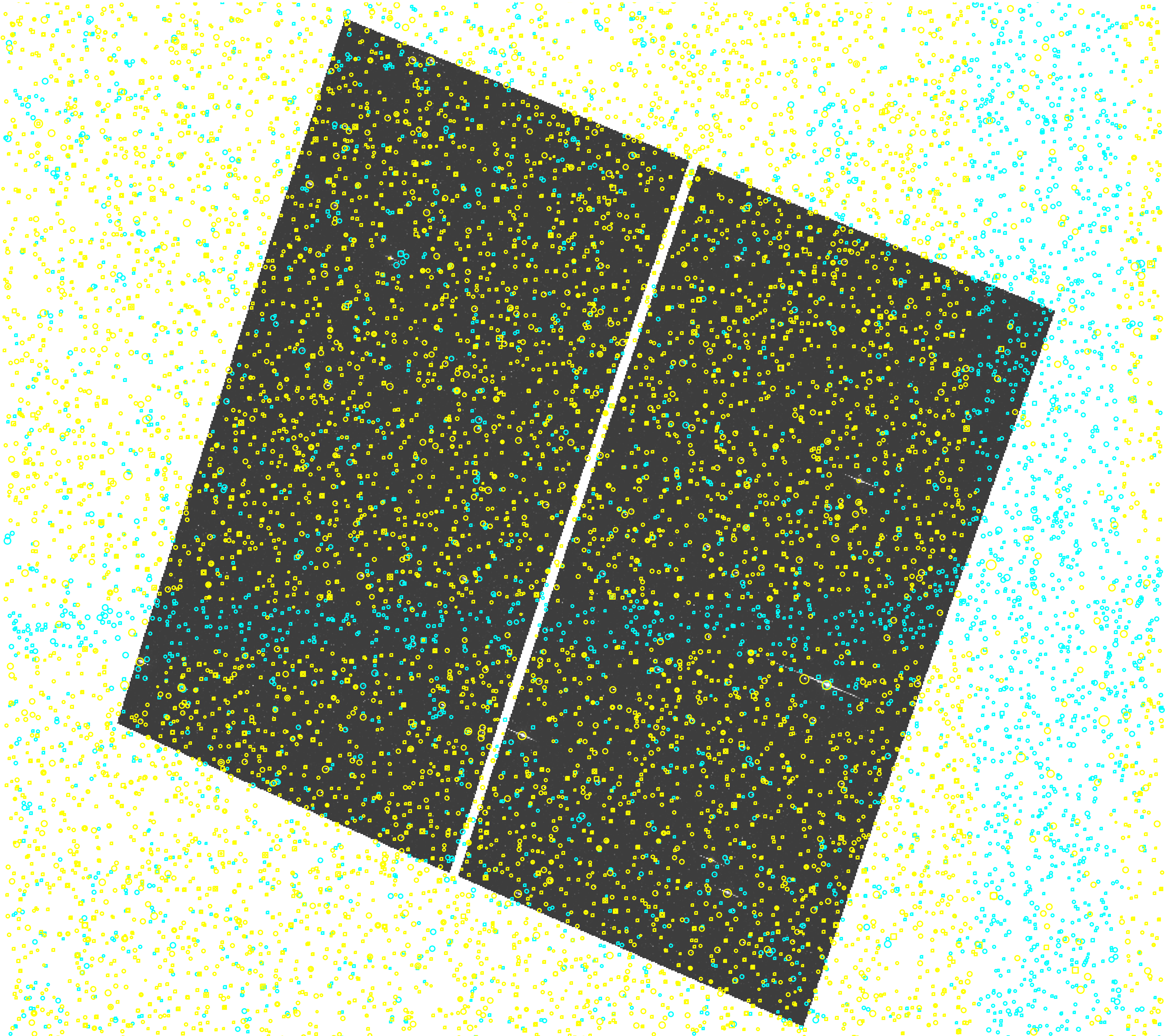

Gaia DR1 is aligned with the International Celestial Reference Frame through Gaia's observations of ~2000 faint (17-20 mag) quasars with accurate VLBI positions.

Gaia's observations show:

(1) excellent agreement between radio and optical positions (RMS < 1 mas)

(2) that the Hipparcos reference frame rotates wrt QSOs by 0.24 mas/yr

# Secondary solution: Reality check on new sources (overlay on HST image - in Baade's Window)

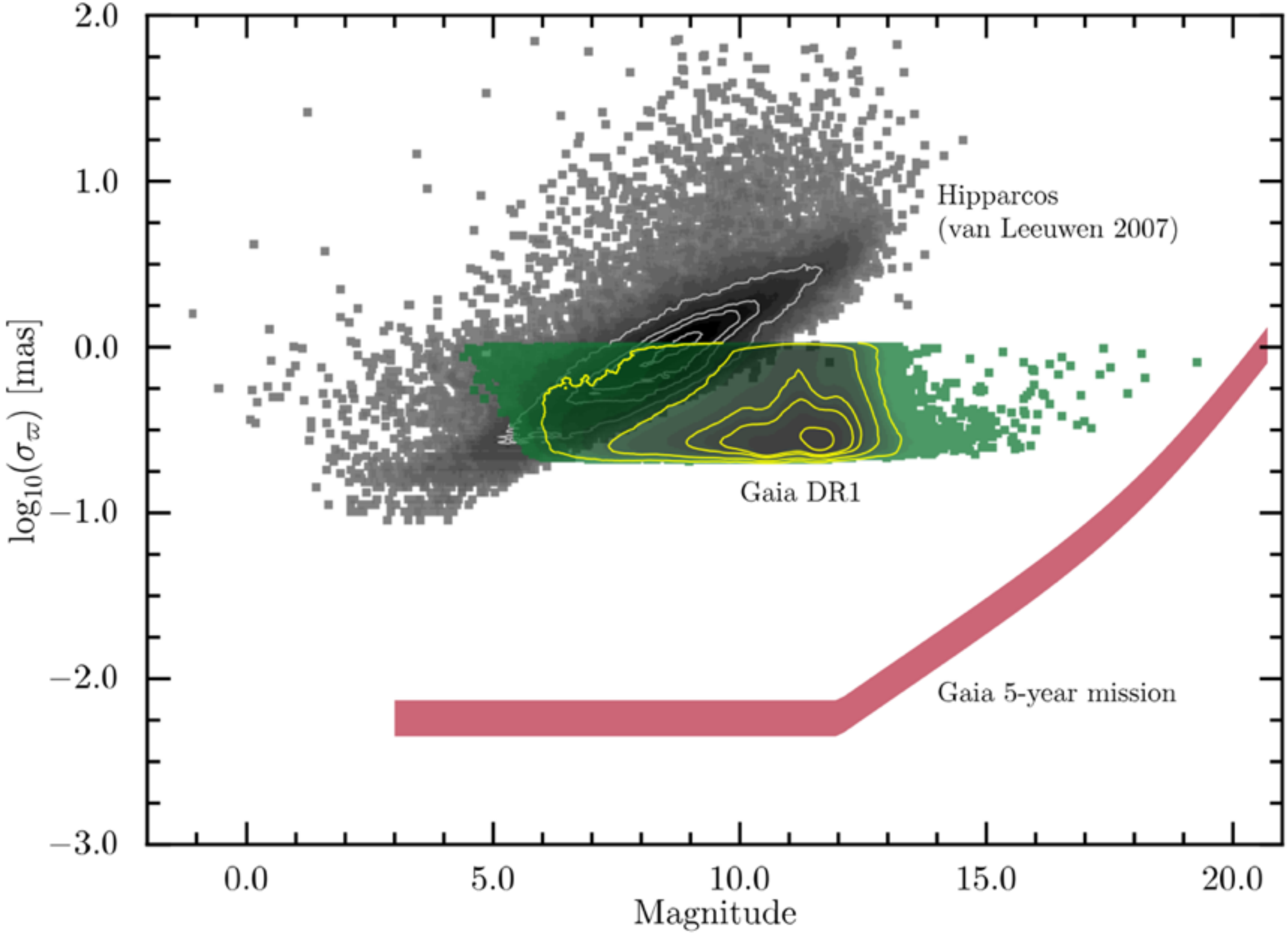


Yellow = IGSL (input list)

Blue = new

# What can be expected from Gaia DR2?

- Will be completely independent of Hipp/Tycho-2
- Based on a longer stretch of data (22 versus 14 months)
- Improved attitude and instrument models will reduce the modelling errors and hence both random and systematic errors in results
- Parallax accuracies of about 50 μas can be reached for sources down to G ~ 15 mag, larger errors for fainter sources


2.0
1.0
0.0
−1.0
−2.0
−3.0
$\log_{10}(\sigma_{\varpi})$ [mas]
Hipparcos
(van Leeuwen 2007)
Gaia DR1
Gaia 5-year mission
0.0
5.0
10.0
15.0
20.0
Magnitude

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- Parallax accuracies of about 50 μas can be reached for sources down to G ~ 15 mag, larger errors for fainter sources
- Proper motions of about 100 μas $yr^{-1}$ (comparable to the Hipparcos subset of TGAS) down to G ~ 15 mag
- This will be obtained for many tens of millions of sources
- Improved and more photometry (G, BP, RP) will enhance the scientific usefulness enormously
- Gaia DR1 is a good training set to get prepared for the real thing!